Rigorous mathematics examination
to achieve the PhDr. degree

Rigorous examination topics – Didactics of Mathematics:

1. Mathematics and didactics of mathematics as scientific disciplines. Position of didactics of mathematics in the structure of educational disciplines, relation to mathematics, education and other sciences. Subject and methods of didactics of mathematics. Development of didactics of mathematics as a scientific discipline.

2. Mathematics as a component of secondary school curricular documents. School mathematics in educational programmes and Framework educational programmes. Didactic analysis of the content of mathematics in secondary school.

3. Modernization of mathematics education. Overview of the development of teaching mathematics in our schools. Basic developmental trends - reformative education, the so-called set mathematics. Foreign ideas and experiences.

4. Basic mathematical concepts. Structure of mathematical knowledge of a secondary school student. Definitions of mathematical concepts, structures and types of definitions. Axioms, theorems, proofs of mathematical theorems.

5. Transmissive and constructivist approaches to mathematical education. Class climate and atmosphere in teaching mathematics. Teacher - student interaction in the process of concept creation. Didactic transformation of mathematical concepts in elementary school.

6. Motivation in mathematics classes with respect to various levels and types of schools. Sources, forms and tools of motivation (didactic game, project).

7. Creativity in mathematics. Creative teacher and creative student. Development of thinking in teaching mathematics. Tasks and means to develop creativity.

8. Communication in teaching mathematics. Language of mathematics and language of school mathematics. Terminology and symbolism in phylogeny and ontogeny. Concept and term.

9. Working methods and procedures in teaching mathematics. Induction, deduction, analogy, experiment, heuristics, algorithm. Didactic principles in teaching mathematics. Clarity, abstraction and generalization.

10. Mathematical learning tasks, their position in teaching mathematics with respect to various levels and types of schools. Didactic functions, types, methods of solution. Working with mathematical learning tasks in the classroom as a reflection of professional and psycho-didactic teacher competences.

11. Evaluation in mathematics. Evaluation of student academic achievement. Diagnostic and prognostic aspect of evaluation. Procedures of obtaining information for evaluation. Incorrect student performance in mathematics classes, its analysis and interpretation.

12. Didactic resources for teaching mathematics. Mathematics textbooks, their functions and parameters. Didactic qualities of mathematics textbooks – examples from school practice. Teaching aids in mathematics.

13. Information and communication technology Didactic use of the calculator and computer. Multimedia resources, interactive systems and technology in teaching mathematics.

14. Mathematical education of talented and handicapped children. Education of mathematical talents, mathematical competitions. Failure In mathematics. Learning disorders and disorders of mathematical skills. Diagnostics and reeducation.

15. History of mathematics as a science. Basic periodization of the history of mathematics, description of individual developmental periods. Use of the history of mathematics in teaching mathematics.

<u>Recommended literature for Didactics of mathematics:</u>

1) Hejný, M. a kol.: Teória vyučovania matematiky 2. Bratislava: SPN, 1990.
2) Hejný, M. – Kuřina, F.: Dítě, škola a matematika. Praha: Portál, 2001.
3) Kuřina, F.: Umění vidět v matematice. Praha: SPN, 1989.
4) Vopěnka, P.: Rozpravy s geometrií. Praha: Panorama, 1989.
5) Kuřina, F.: Deset pohledů na geometrii. Praha: ALBRA, 1996.
6) Zelina, M.: Tvořivost v matematice. Ostrava: KPÚ, 1990.
7) Kalhous, Z., Obst, O.: Školní didaktika. Praha: Portál, 2002.
8) Petty, G.: Moderní vyučování. Praha: Portál, 1996.
9) Bruner, J. V.: Vzdělávací proces. Praha: SPN, 1965.
10) Kopka, J.: Hrozny problémů v matematice. Ústí n. L.: 1998.
11) Odvárko, O.: Metody řešení matematických úloh. Praha: SPN, 1990.
12) Fulier, J., Šedivý, O.: Motivácia a tvorivosť v matematike. Nitra: UKF, 2001.
13) Květoň, P.: Kapitoly z didaktiky matematiky 1, 2. Ostrava: 1990.
14) Novotná, J.: Analýza řešení slovních úloh. Praka: UK, 2000.
15) Plocki, A.: Pravděpodobnost kolem nás. Ústí n. L.: 2001.
16) Novák, B.: Matematika III. Několik kapitol z didaktiky matematiky. Olomouc: VUP, 1999.
17) Opava, Z.: Matematika kolem nás. Praha: Albatros, 1989.
18) Kárová, V.: Počítání bez obav. Praha: Portál, 1996.
19) Sedláčková, J.: Diagnostické metody ve vyučování matematice. Olomouc: VUP, 1993.
20) Krejčová, E. – Volfová, M.: Inspiromat matematických her. Hradec Králové: Gaudeamus, 1994.
21) Kittler, J.: Tři cesty k porozumění matematice. Komenský, r. 117, 1992/93, č. 1
22) Divíšek, J. a kol.: Didaktika matematiky pro studium učitelství 1. stupně ZŠ. Praha: SPN, 1989.
23) Mikulčák, J.: Didaktika matematiky 1. Praha: SPN, 1982.
24) Gábor, O., Kopaněv, O. Križalkovič, K. Teória vyučovania matematiky 1. Bratislava: SPN, 1989.
25) Růžička, E., Růžičková, B.: Technologie vzdělávání. Olomouc: UP, 1998.
26) Novák, B., Stopenová, A.: Slovní úlohy ve vyučování matematice na 1. Stupni ZŠ. Olomouc: UP, 1993.
27) Struik, D. J.: Dějiny matematiky. Praha: Orbis, 1963.
28) Vyšín, J.: Štyry kapitoly o problémovom vyučovaní matematiky. Bratislava: SPN, 1978.

Rigorous examination topics – Algebra:

1. Basic knowledge about propositions and sets – proposition, compound proposition, logical connective, proposition form (predicate), quantifiers, basic types of proofs in mathematics. Set and its determination, set associations and operations, Venn diagrams.

2. Binary relations over sets and their properties - Cartesian product of sets, binary relations over sets and their properties, equivalence relation, sets decomposition, order relation, function, injective function, bijection.

3. Algebraic structures with one binary operation – binary operations over sets and their properties, algebraic structures with one binary operation – groupoid, semigroup, group (specific examples). Substructure of algebraic structure, homomorphism and isomorphism of structures.

4. Algebraic structures with two binary operations – ring, integral domain, object (specific examples). Substructure of algebraic structure, homomorphism and isomorphism of structures.

5. Vector spaces – vector spaces over subjects (specific examples), subspace of vector space, linear set span, bases and dimensions of vector space, homomorphism and isomorphism of vector spaces, base vector coordinates.

6. Matrices and determinants – Matrices, basic operations with matrices, structure of matrices and their properties, rank and determination of matrices, determinant of square matrices and its calculation, inverse matrices and their calculation. Use of matrices and determinants in solving systems of linear equations – Frobenius theorem, Cramer's rule, homogeneous system and its solutions.

7. Divisibility in the integral domain – 'division' relation and its properties, associated members, non-trivial and trivial divisors, irreducible members (specific examples of these concepts), greatest common divisor and least common multiple and their calculation, incommensurable members. Divisibility of integers – prime and composite numbers, decomposition of an integer into a product of prime numbers and its uniqueness.

8. Polynomials – polynomials above numeral ring, equality of polynomials, sum and product of polynomials, structure of polynomials and its properties. Polynomial root, root multiplicity, polynomial decomposition into a product of irreducible polynomials.

9. Algebraic equations – Algebraic equations, algebraic solvability, equations of the 1st to 4th grade and their algebraic solvability, insolvability of higher degree equations (historical development), binomial equations, reciprocal equations. Numerical methods of equation solving, bisection method, chord method, tangent method.

<u>Recommended literature for Algebra:</u>

1. BLAŽEK, J. a kol. Algebra a teoretická aritmetika 1. a 2. Praha: SPN, 1983, 1985.
2. BURRIS, S., SANKAPPANAVAR, H.P. A Course in Universal Algebra. Springer-Verlag, 1981.
3. EMANOVSKÝ, P. Algebra 2 (pro distanční studium). Olomouc: VUP, 2001.
4. EMANOVSKÝ, P. Cvičení z algebry (polynomy, algebraické rovnice). Olomouc: VUP, 1998.
5. EMANOVSKÝ, P. Algebra 3 (pro distanční studium). Olomouc: VUP, 2002.
6. EMANOVSKÝ, P. Cvičení z algebry (algebraické struktury). Miniskriptum, PdF UP Olomouc, 1993.
7. Emanovský, P. Algebraické struktury ve vysokoškolské přípravě učitelů matematiky. Olomouc : VUP, 2000.
8. KOPECKÝ, M., EMANOVSKÝ, P. Sbírka řešených příkladů z algebry. Olomouc : VUP, 1990.
9. KOPECKÝ, M. Základy algebry. Olomouc: VUP, 1998.
10. KOŘÍNEK, V. Základy algebry. Praha : NČSAV, 1956.
11. KRUTSKÝ, F. Algebra I. Olomouc : VUP, 1995. PřF UP Olomouc, 1998.
12. MALCEV, A. I. Algebraičeskije sistemy. Moskva: Nauka. 1970.
13. PROSKURJAKOV, I. V. Sbornik zadač po linejnoj algebre. Moskva: GIFML, 1962.
14. Schwarz, Š. Základy náuky o riešení rovníc. Praha: NČSAV, 1958

Rigorous examination topics – Geometry:

1. Vector spaces. Subspaces of a vector space, linear transformations of vector spaces, characteristic vectors, vector space orientation.

2. Affine spaces. Arithmetic and geometric model of an affine space, transformation of coordinates in an affine space, subspaces, their relative positions and their parametric and non-parametric expression.

3. Affine transformation. Properties, numerical expression, associated transformation and its matrix. Affinity of space, subspace transformation. Invariant points, characteristic equation, characteristic vectors and invariant directions. Parallel projection of affine space into hyperplane. Basic affinity and its numerical expression, elation. Decomposition of affinity into basic affinities, direct and indirect affinity. Affine group and its subgroups.

4. Order and its properties. Dividing ratio of points. Order on a line, half-space and its parametric and non-parametric expression. Layer, wedge, reflex angle. Parametric expression of a reflex angle and triangle. Convex sets.

5. Euclidean vector space. Scalar product, vector size, vector perpendicularity. Orthogonal and orthonormal basis, Schmidt orthogonalization process, scalar product in space with orthonormal basis, orthogonal subspace complement, orthogonal projection. Perpendicularity, total perpendicularity of subspaces. Normal vector of hyperplane, line perpendicular to hyperplane, perpendicularity of hyperplanes. Determination of totally perpendicular subspace to a given subspace.

6. Euclidean point space. Distance between points, introduction of metrics, Cartesian coordinate system, orthonormal matrix and its properties. Distance between subspaces. Perpendicular to a subspace. Distance between a point subspace and hyperplane. Distance between parallel and skew subspaces.

7. Subspace deviations. Deviation of vectors and one-dimensional subspaces. Definition of deviation of non-trivial subspaces, determination of deviation. Deviation of two lines, line and hyperplane, line and subspace, two hyperplanes.

8. Isometrics. Symmetry by hyperplane, translation, symmetry by centre, classification of identical transformations in E1, classification of identical transformations in E2, classification of identical transformations in E3.

9. Similar transformations. Properties, associated transformation, its matrix and properties. Group of similarities and its subgroups. Invariant points and similarity directions, characteristic roots and vectors. Homothety, numerical expression, properties. Monge's theorem on homothety composition, Monge's groups. Decomposition of similarity to homothety and congruence. Classification of similarities in E1 and E2.

<u>Recommended literature for Geometry:</u>

1. MATYÁŠEK, F. Geometrie. 1. vydání. Olomouc: UP, 1995.
2. SEKANINA, M. a kol. Geometrie 1. Praha: SPN, 1986.
3. HORÁK, P., JANYŠKA, J. Analytická geometrie. Brno: MU, 1997.

Rigorous examination topics – Mathematical analysis:

1. Solid of revolution
Calculation of the volume of a solid of revolution using the Riemann integral. Definition of the Riemann integral.

2. Plane figure
Riemann integral and its application for the calculation of the area of a plane figure.

3. Basic methods used in integral calculus
Special emphasis on integration of a rational fraction function.

4. Real function of a real variable
Examining the behaviour of the function in a rectangular Cartesian coordinate system.

5. Taylor and Maclaurin formula
Use of the Taylor and Maclaurin formula. Taylor and Maclaurin series of functions of a single real variable. Development of elementary functions.

6. Differentiation of a function
Definition of differentiation of a function in a point and interval. Geometric and physical meaning. Formulas for the differentiation of elementary functions.

7. Sequences and series of real numbers
Limit of the sequence, sum of a series, arithmetic and geometric series.

8. Extremes of a real function of a single real variable
Methods of determination of extremes of a function of a single real variable.

9. Definition of the e number and the concept of the natural logarithm

<u>Recommended literature for Mathematical analysis:</u>

1. JARNÍK, V.: Diferenciální počet I. Praha: Academia, 1984.
2. JARNÍK, V.: Integrální počet I. Praha: Academia, 1984.
3. ŠKRÁŠEK. J., TICHÝ, Z.: Základy aplikované matematiky I. Praha: SNTL, 1983.
4. ŠKRÁŠEK, J., TICHÝ, Z.: Základy aplikované matematiky II. Praha: SNTL, 1986.

Rigorous examination topics – Arithmetic and the number theory:

1. Peano axioms of natural numbers. Induction axiom and its significance. Operations over a set of natural numbers and their properties. Ordering of N set. Divisibility features in the decimal system. Systems of z base. Operations in systems of z bases.

2. Prime numbers. Properties of prime numbers, infinity of a set of prime numbers. Use of prime numbers in didactic applications. Structures with unique decomposition. Fermat and Mersenne prime numbers.

3. Construction of the integrity domain of integers. Construction of Z set, structure (Z, +), isomorphic embedding (N, +) to (Z, +), structure (Z, +), structure (Z, .) and its properties. Ordering of the integrity domain of integers.

4. Construction of a field of rational numbers. Construction of a field of fractions to integrity domain (Z, +,.). Isomorphic embedding (Z,.) to (Q,.). Issues related to ordering. Fraction of a rational number. Expressing rational numbers in systems of z base.

5. Congruence equations of the first degree. Solving congruence equations of the first degree using the Euler's theorem. Continued fractions. Solving congruence equations of the first degree using continued fractions. Use of congruence equations of the first degree in solving Diophantine equations.

6. Construction of a field of real numbers. Dedekind cut theory, rational and irrational cuts, algebraic operations over a set of cuts. Issues of continuity.

7. Approximation of real numbers by rational numbers. Decimal fractions, infinite development of rational numbers, concept of approximation. Irrational numbers as limits of sequences. Use of the Monte Carlo method for approximation of irrational numbers. Continued fractions of real numbers and their properties.

8. Construction of a field of complex numbers. Properties of a field of complex numbers, algebraic and goniometric form, operation of numbers in a goniometric form, De Moivre's formula. Inability to order a field of complex numbers. Isomorphic embedding of a field (R,+ ,.) to (K,+ ,.). Groups of complex nth roots.

9. Algebraic and transcendental numbers. Roots of algebraic equations, association with Euclidean structures. Irrational numbers. Liouville's theorem, transcendental numbers.

<u>Recommended literature for Arithmetic and the number theory:</u>

1. BLAŽEK, J. a kol. Algebra a teoretická aritmetika 1. Praha: SPN . 1983.
2. HARDY, G., WRIGHT, E.M. An introduction to the theory of numbers. Oxford: Gibbs´edition, 1965.
3. LUGOWSKI, H. a kol. Einführung in die Algebra, Arithmetik und Zahlentheorie. Potsdam: Pädagogische Hochschule. 1965.
4. KOPECKÝ, M. Aritmetika. Olomouc: Univerzita Palackého. 2002.
5. HALAŠ, R. Teorie čísel. Olomouc: Univerzita Palackého. 1997.